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An atmospheric model for numerical integration including the tropopause effects

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AIR MINISTRY

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1. Introduction

The atmospheric models which have been used for the numerical integration of the equations of motion have divided the atmosphere into slices bounded by isobaric surfaces and these models may be regarded as either ignoring the tropopause or having a fixed tropopause. It is very convenient mathematically to use such divisions since the equations used for forecasting are expressed in terms of pressure rather than of height; they are obtained either by direct expression in finite differences, as in the Charney-Phillips (1953) hierarchy of models, or by averaging in the vertical between two fixed pressure levels, as in the Sawyer-Bushby (1953) and similar models. Physically, however, it is clear that the atmosphere is divided into two parts with different physical characteristics, and especially with different stabilities, above and below the tropopause which varies in pressure from about 100 mb to 500 mb. The effect of the stable upper layer will be to reduce the intensity of the disturbances which are found in the lower layers and it is reasonable to think that the height of the boundary will aid in determining the reduction.

The following analysis is concerned with a simple model including a variable pressure surface which is identified with the tropopause. It is assumed that the vertical velocity has a parabolic profile with pressure in both the troposphere and stratosphere; this profile is determined by two parameters, the pressure and vertical velocity at the tropopause. The divergence is thus a linear function of pressure in both stratosphere and troposphere and its pressure derivative has a discontinuity at the tropopause, consistent with the jet stream and a change in static stability. This simple assumption about the vertical velocity shows some of the major requirements of a physical definition of a tropopause. The thermal wind is taken as a linear function of pressure and constant in direction, so that the results of time integrations should then be comparable with the results obtained by using the Sawyer-Bushby model.

2. The model

The model is represented pictorially in Fig.1 which explains the notation. H_0, H_1 and H represent heights above the mean sea level; p_0 and p_1 are two pressure levels at which dp/dt vanishes, while P is a fixed pressure surface; h' is the thickness between p_0 and P. The tropopause pressure is p_T and at that pressure $dp/dt = \omega_T$; it is assumed that $\omega = dp/dt$ at any given level p is given by

$$\omega = \omega_T \alpha(p, p_T) \quad , \quad p_1 \leq p \leq p_T$$
$$= \omega_T \beta(p, p_T) \quad , \quad p_T \leq p \leq p_0 \qquad \ldots\ldots(1)$$

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In order to specify the form of the functions α and β we assume that there is a linear change of isobaric divergence with pressure in both the troposphere and the stratosphere, vanishing at the upper level p_1. The thermal wind shear is taken to be linear in the troposphere, and in the stratosphere if predictions are to be extended to levels above the tropopause. The equations written here will apply to the troposphere but the extension to the stratosphere is clear. It will be observed that once again we are making the crudest assumptions which fit the general observations, following the practice of taking the simplest models possible in a first examination of the problem.

3. The variation of vertical velocity and wind with pressure

Equation (1) is sufficient for our purpose, but with the assumption of a linear distribution of isobaric divergence with pressure we have

$$\omega = \omega_T \left(\frac{p - p_1}{p_T - p_1}\right)^2, \qquad p_1 \leq p \leq p_T$$

$$= \omega_T \left[2\left(\frac{1}{p_T - p_0} - \frac{1}{p_T - p_1}\right)(p - p_0) + \left(\frac{2(p_T - p_0)}{p_T - p_1} - 1\right)\left(\frac{p - p_0}{p_T - p_0}\right)^2 \right]$$

$$p_T \leq p \leq p_0 \qquad \ldots\ldots (2)$$

If we take $p_0 = 1000$ mb, $p_1 = 0$ mb, then from equation (2)

$\omega_T = (9/25)\,\omega_m$ $\qquad p_T = 200$ mb

$= \frac{3}{4}\,\omega_m$ $\qquad p_T = 500$ mb

where ω_m is the maximum value of ω, showing that in dealing with ω_T we are not dealing with a quantity which is an order of magnitude less than the maximum or mean vertical velocity.

The variation of wind with pressure is expressed in terms of the thermal wind between two fixed levels p_0, P below the tropopause

$$V = V_0 + \frac{p - p_0}{P - p_0} V' \qquad p_T \leq p \leq p_0 \qquad \ldots\ldots (3)$$

$$\left.\begin{aligned} V' &= k \wedge \frac{g}{f} \nabla h' \\ V_0 &= k \wedge \frac{g}{f} \nabla h_0 \end{aligned}\right\} \qquad \ldots\ldots (4)$$

with similar vorticity equations.

4. The adiabatic equation

We assume that the motion is adiabatic, although non-adiabatic effects can be allowed for. The equation for the thickness h' between the levels p_0 and P is

$$\frac{\partial h'}{\partial t} + V.\nabla h' = -\frac{R}{g}\int_{p_0}^{P} \omega\left(\frac{\gamma}{g\rho} - \frac{\partial T}{\partial p}\right)\frac{dp}{p} \qquad \ldots\ldots (5)$$

where γ is the suitable adiabatic lapse rate and ρ the density. We require an assumption about the integrand and take $\frac{\gamma}{g\rho} - \frac{\partial T}{\partial p} = -K$ where K is a constant evaluated from the mean values obtained by observation. Then,

$$\frac{\partial h'}{\partial t} + V.\nabla h' = \frac{R}{g} K \omega_T \, G(p_0, P, p_T, p_1) \qquad \ldots\ldots (6)$$

where

$$G_1 = \int_{p_0}^{P} \frac{dp}{p}\left[\begin{array}{l} 2\left(\frac{1}{p_T - p_0} - \frac{1}{p_T - p_1}\right)(p - p_0) + \\ \left\{2\frac{(p_T - p_0)}{p_T - p_1} - 1\right\}\left(\frac{p - p_0}{p_T - p_0}\right)^2 \end{array}\right]$$

and is easily evaluated. Taking the Laplacian of (6)

$$\frac{\partial}{\partial t}\nabla^2 h' + \nabla^2(V.\nabla h') = \frac{KR}{g}\nabla^2(G\omega_T) \qquad \ldots\ldots (7)$$

and allowance could be made in this equation for space variations of K.

5. The diagnostic equation for ω_T

Neglecting the twisting terms the vorticity equation is

$$\frac{\partial \zeta}{\partial t} + V\nabla(\zeta + f) + \omega\frac{\partial \zeta}{\partial p} = (\zeta + f)\frac{\partial \omega}{\partial p} \qquad \ldots\ldots (8)$$

and subtracting that at the level p_0 from that at the level P

$$\frac{\partial \zeta'}{\partial t} + V_0 \cdot \nabla \zeta' + V' \cdot \nabla (\zeta_0 + \zeta' + f) =$$

$$\omega_T (2\zeta_0 + \zeta' + 2f)\left(\frac{P-p_0}{p_T-p_0}\right)^2 \left(2\frac{(p_T-p_0)}{p_T-p_1} - 1\right) \qquad \ldots\ldots (9)$$

where ω and derivatives have been expressed in terms of ω_T. If we now assume that the winds are sufficiently geostrophic

$$\zeta' = \frac{g}{f} \nabla^2 h'$$

so that combining equations (7) and (9)

$$\frac{KR}{g} \nabla^2 (G \omega_T) - \frac{f \omega_T}{g} (2\zeta_0 + \zeta' + 2f)\left(\frac{P-p_0}{p_T-p_0}\right)^2 \left(2\frac{p_T-p_0}{p_T-p_1} - 1\right) =$$

$$\nabla^2 (V \nabla h') - \frac{f}{g} V' \cdot \nabla (\zeta_0 + \zeta' + f) - \frac{f}{g} V_0 \cdot \nabla \zeta' \qquad \ldots\ldots (10)$$

Having eliminated the time variable, equation (10) is diagnostic in ω_T and can be solved either by integration or by finite difference methods.

6. Forecast of thickness and height of an isobaric surface

The thickness tendency equation to the level P is

$$\frac{\partial h'}{\partial t} + V \cdot \nabla h' = \frac{R}{g} K G \omega_T \qquad \ldots\ldots (11)$$

and this equation is used as a prognostic equation in h' since ω_T has already been obtained from equation (10). It remains to obtain a forecasting equation for H_0 or for H. Owing to the proximity to the surface it is possible that the vorticity equation

$$\frac{\partial}{\partial t} \frac{g}{f} \nabla^2 H_0 + V_0 \cdot \nabla \left(\frac{g}{f} \nabla^2 H_0 + f\right)$$

$$= 2\left(\frac{g}{f} \nabla^2 H_0 + f\right)\left(\frac{1}{p_T-p_0} - \frac{1}{p_T-p_1}\right)\omega_T \qquad \ldots\ldots (12)$$

/is

is unsuitable although it should be tried. Otherwise we solve the corresponding vorticity equation at the level P for H and subtract the thickness to obtain H_o .

7. Forecast of tropopause pressure

We note that

$$\omega_T = \frac{dp_T}{dt} = \frac{\partial p_T}{\partial t} + V_T . \nabla p_T$$

so that

$$\frac{\partial p}{\partial t} = \omega_T - \left(V_o + \frac{p_T - p_o}{P - p_o} V' \right) . \nabla p_T \qquad \ldots\ldots (13)$$

which is the prognostic equation for p_T .

References

(1) Charney, J.C. and Phillips, N. 1953 J.Met., 10, p. 71.

(2) Sawyer, J.S. and Bushby, F.H. 1953 J.Met., 10, p. 54.

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Pictorial representation of the model

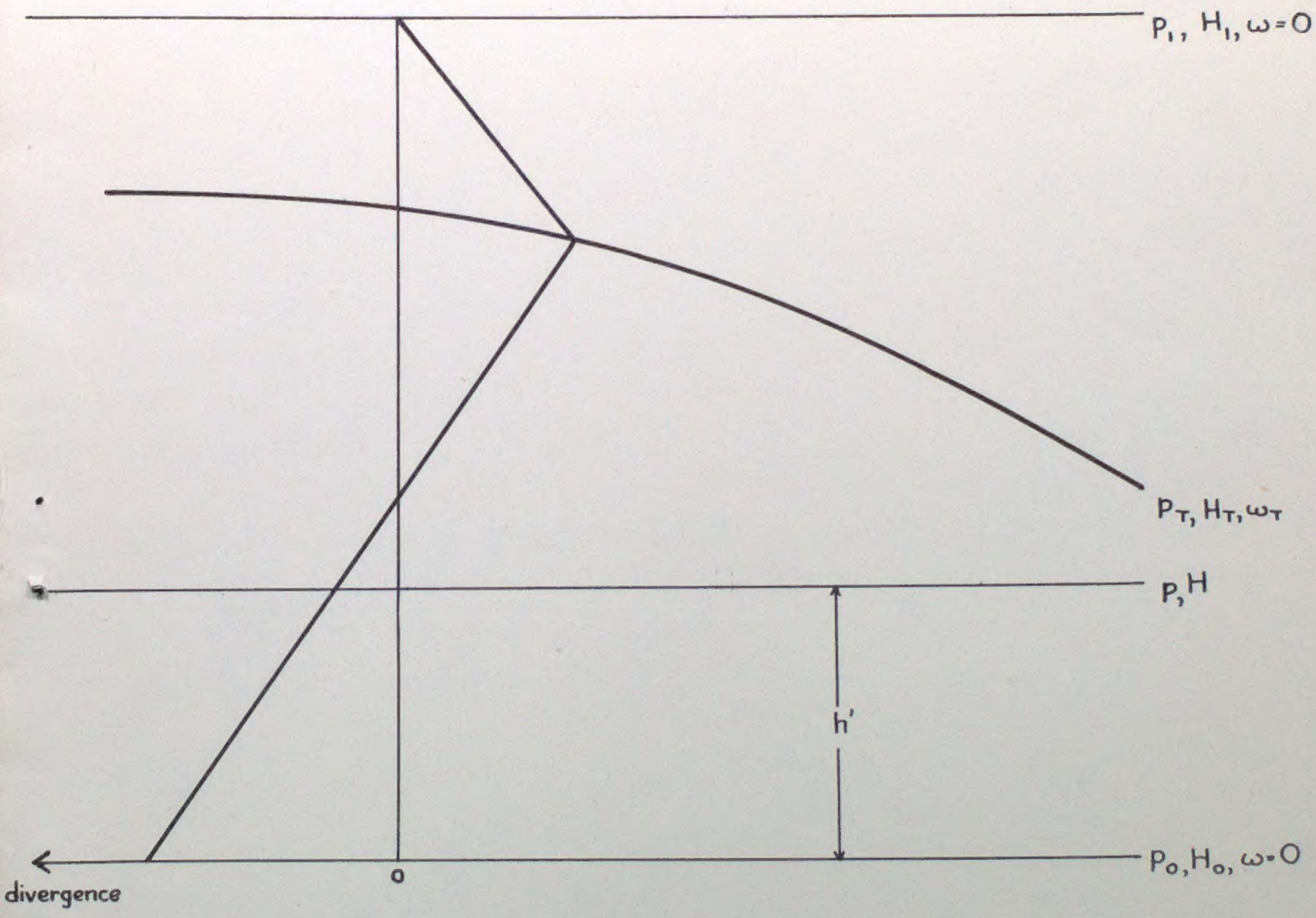

Fig. 1